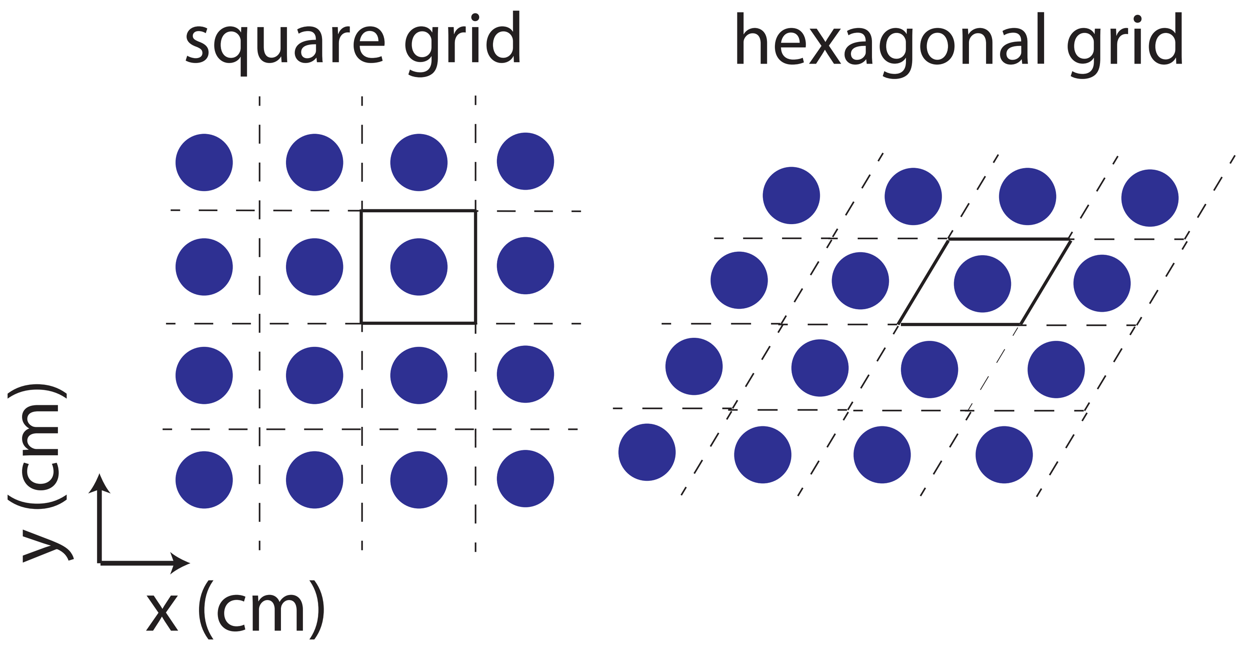
Encoding multiple spaces in grid-cells networks

Alexis Dubreuil[1], Rémi Monasson[1], Alessandro Treves [2]
1. Laboratoire de Physique Théorique, Ecole Normale Supérieure, Paris, France
2. Cognitive Neuroscience, SISSA, Trieste, Italy

Grid-cells found in enthorinal cortex form a representation of space through their receptive fields that pave space with hexagonal lattices. It has been proposed that the firing of grid-cells with similar grid spacing is supported by recurrent connectivity, such that network dynamics lies on a 2-D manifold of stable states [1]. Recent empirical evidence suggests that the grid-cell code is not limited to physical space encoding in enthorinal cortex, but could also subserve encoding of more abstract spaces in other cortical areas [2]. In order to assess the efficiency of neural networks using a grid-cell code, the present study is focused on the problem of counting the number of grid manifolds that can be reliably embedded in a neural network.

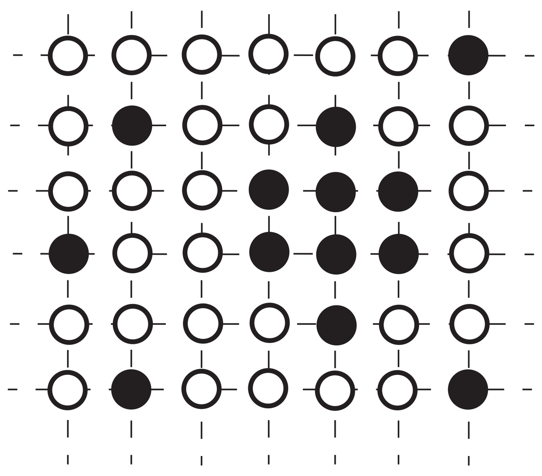
Square grid and hexagonal grid

Receptive field of a cell



Study a primitive cell (square or rhombus) with different periodic boundary conditions

Microscopic description of the network



Binary neurons: $\vec{\sigma} = (\sigma_1, ..., \sigma_N) \in \{0,1\}^N$

Hebbian ynaptic matrix:

$$W_{ij} = \sum_{l=0}^{L} W_{ij}^{l} = W_{ij}^{0} + \sum_{l=1}^{L} W^{0}_{\pi^l(i)\pi^l(j)}$$

$$W_{ij}^{0} = \begin{cases} \frac{1}{N} & \text{if } d_{ij} \leq d_c \\ 0 & \text{if } d_{ij} > d_c \end{cases} \; ; \; d_c = \sqrt{\frac{wN}{\pi}}$$

Distribution of microscopic states:

$$P_W(\vec{\sigma}) = \frac{1}{Z}\exp(\beta \sum_{i<j} W_{ij}\sigma_i\sigma_j) \; ; \; \sum_i \sigma_i = fN$$

Macroscopic description of the network

(SP)

$$\mu(\vec{x}) = \int d\vec{y}\, W(\vec{x}-\vec{y})\rho(\vec{y}) + \lambda$$

$$\rho(\vec{x}) = \int Dz \left[1 + e^{-\beta\sqrt{\alpha r} - \beta\mu(\vec{x})}\right]^{-1}$$

$$q = \int d\vec{x} \int Dz \left[1 + e^{-\beta z\sqrt{\alpha r} - \beta\mu(\vec{x})}\right]^{-2}$$

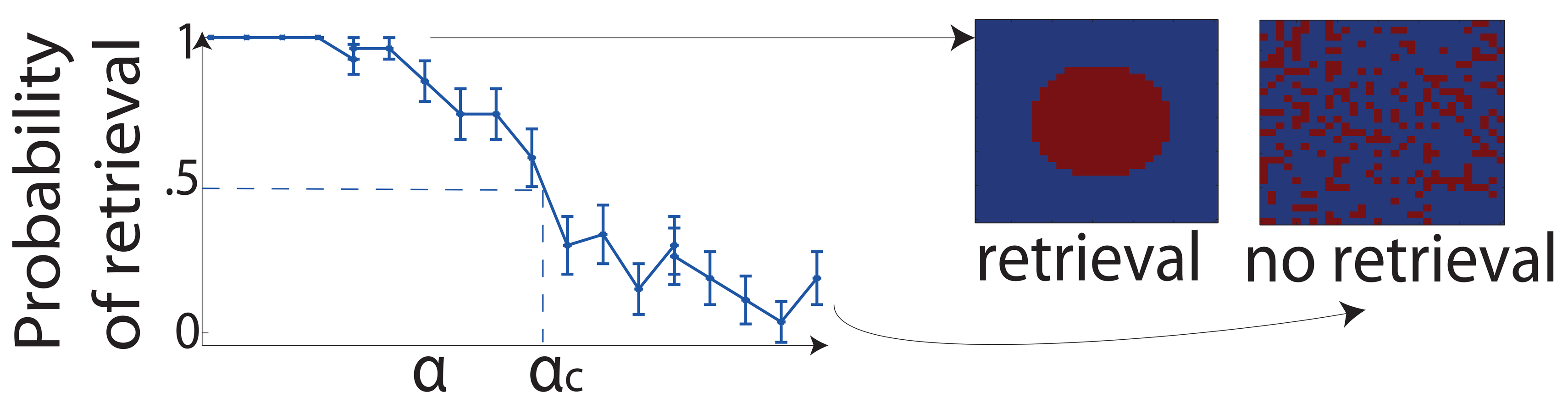
$$r = (q - f^2) \sum_{(k_1,k_2)} [\Phi(k_1,k_2) - \beta(f-q)]^{-2}$$

$$\alpha = \frac{L}{N}$$

Square: $k_1 = p$, $k_2 = q$; Hexagonal: $k_1 = p$, $k_2 = (q - \frac{p}{2})\frac{2}{\sqrt{3}}$; $p, q \in \{0, ..., \sqrt{N} - 1\}$

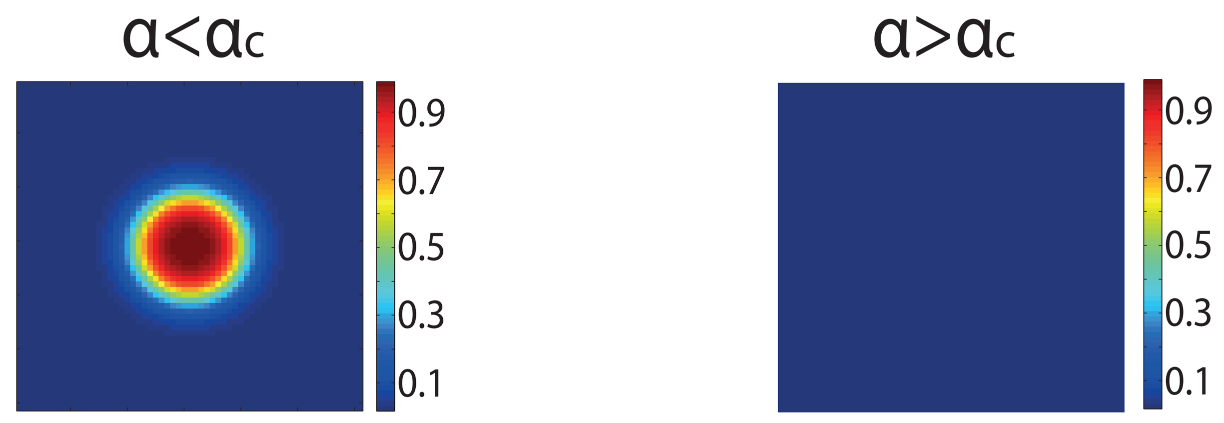
Computing network storage capacity

Simulations: Monte Carlo from the microscopic model



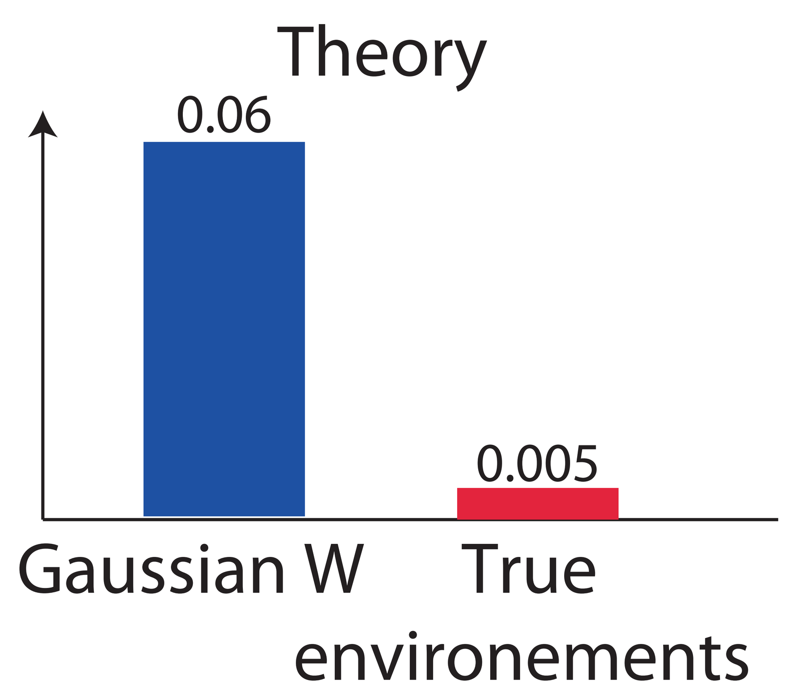
Theory:

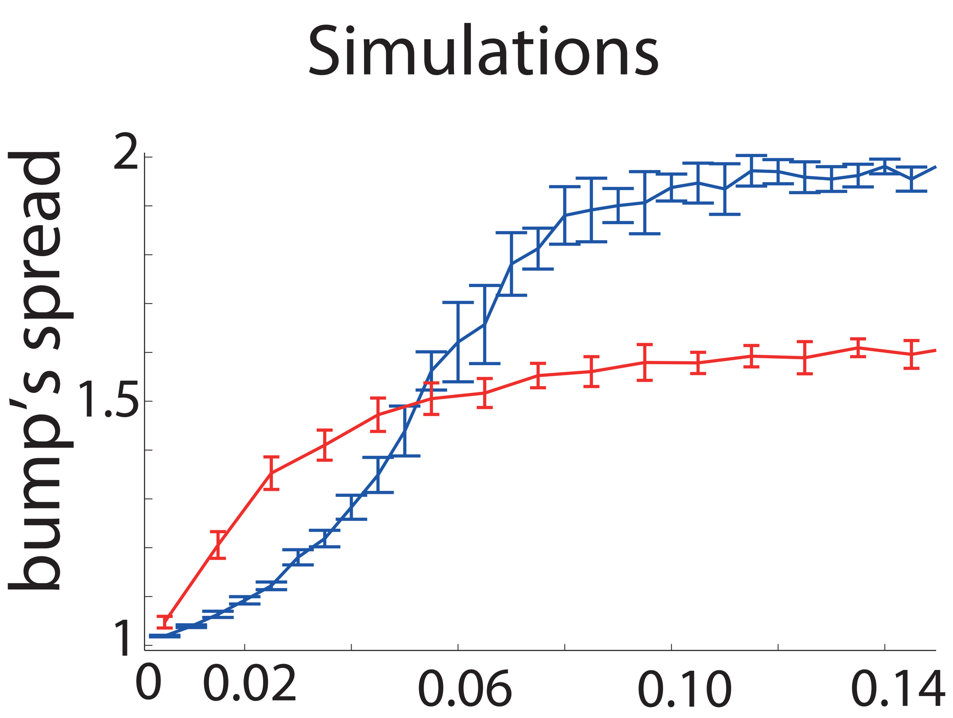
Solve SP, find the largest α such that ρ is still localized



On the importance of the synaptic noise structure

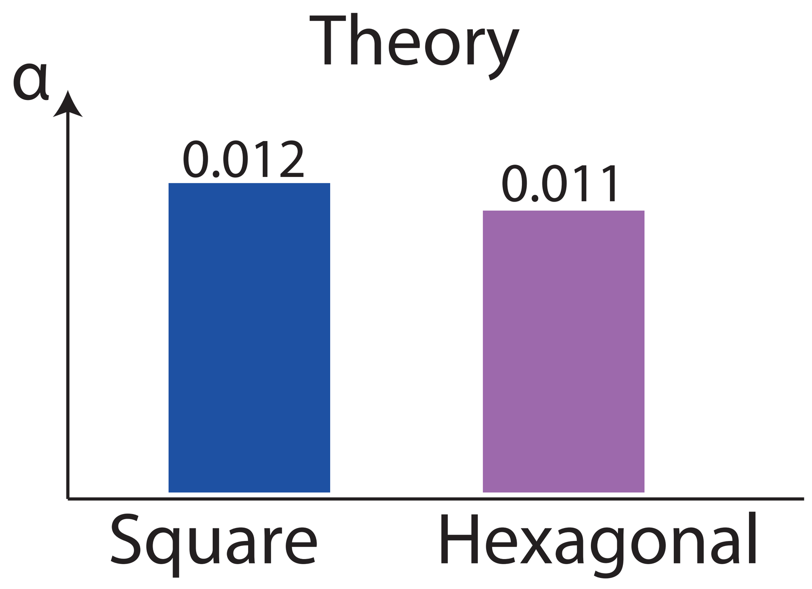
f= 0.25 ; w = 0.25 ; 1/β = 0

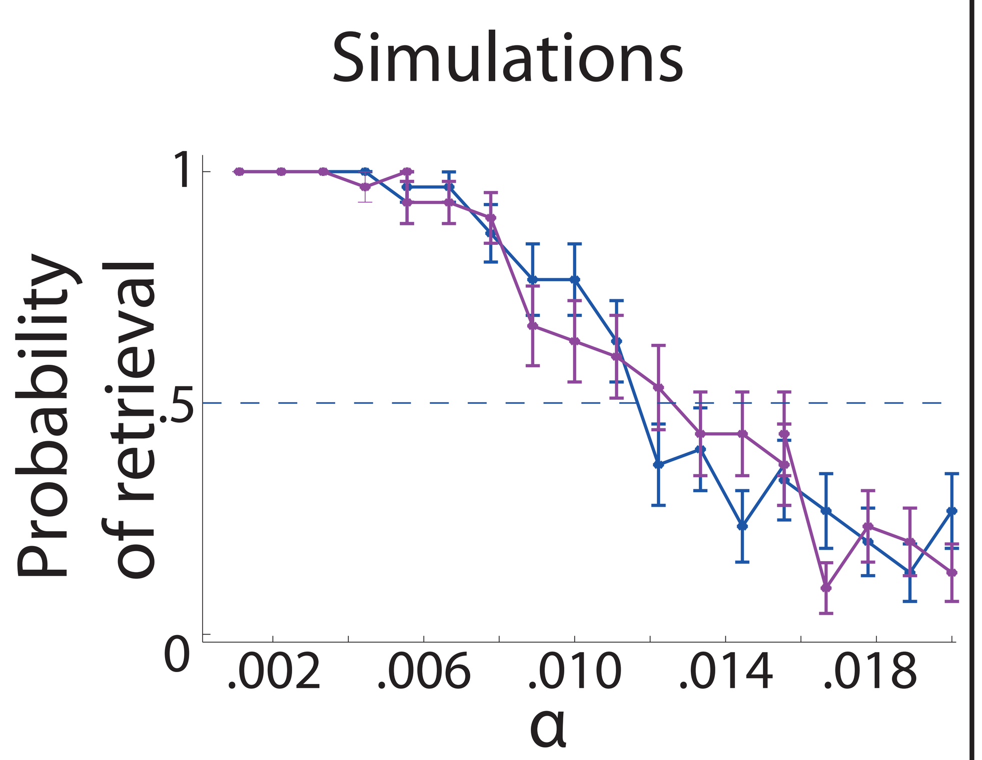




Square VS Hexagonal storage capacity

f= 0.1 ; w = 0.05 ; 1/β = 0.004





References
1. Burak Y, Fiete IR, PLoS Computational Biology 2009.
2. Constantinescu AO, O'Reilly JX, Behrens TEJ, Science 2016.
3. Monasson R, Rosay S, Phys Rev E 2013.

contact: alexis.dubreuil@gmail.com

Conclusions

- ≈100 manifolds can be stored in a grid cell network of 10,000 neurons
- Grid cell networks as good as place cell networks
- With threshold-linear neurons, grid cell networks perform much better (cf poster 274)