Memory capacity of networks with stochastic binary synapses

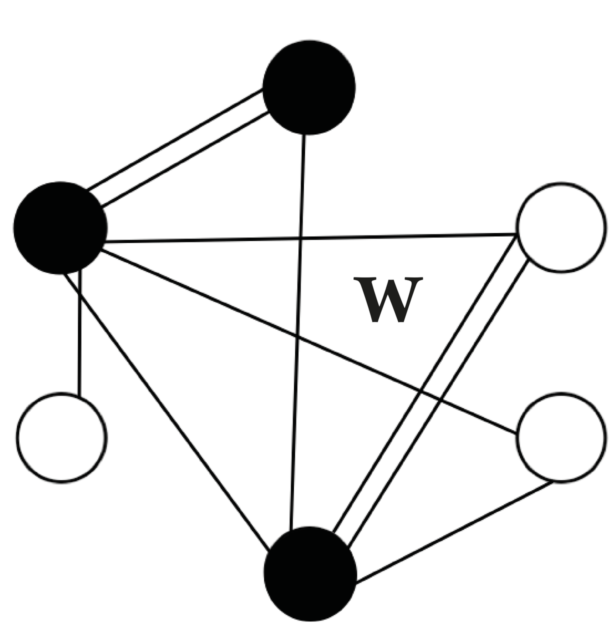
A. Dubreuil[1,2], Y. Amit[2], N. Brunel [2]

1. Université Paris Descartes & CNRS, Brain Physiology lab
2. University of Chicago, departments of Statistics and Neurobiology

Introduction

Lesion studies suggest that episodic memories are initially stored in the hippocampus, but are then transferred to cortex where a long-term memory trace is stored.
This suggests that in the hippocampus, memories have to be acquired in one shot, while in cortex, they are acquired slowly over multiple repetitions. In the present work, we study the memory capacity of networks that have to acquire memories either in one shot, or through multiple presentations.

Network description



Dynamics:

$$\sigma_i(t+1) = \Theta\left(\sum_{j=1}^{N} W_{ij}\sigma_j(t) - fN\theta\right)$$

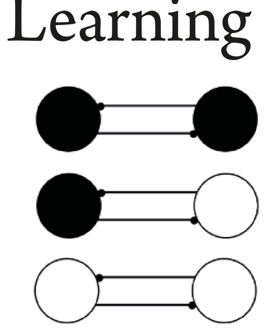
$$\sigma_i \in \{0,1\}$$

$$W_{ij} \in \{0,1\}$$

Memories $\vec{\xi}^{\mu}$, coding level f:

$$\xi_i^{\mu} = \begin{cases} 1 & \text{with probability } f \\ 0 & \text{with probability } 1-f \end{cases}$$

Learning rule:



Potentiate with proba q+

Depress with proba q-

Do nothing

Learning protocols:

(H) New patterns are presented once to the network

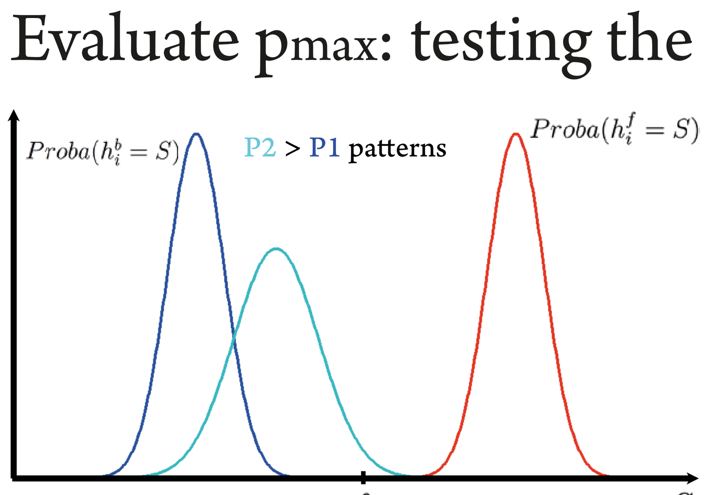
(C) A sequence of P patterns is presented multiple times to the network

Storage capacity calculation (1)

$$i \ln 2 = p_{max} \frac{N(-f\ln f - (1-f)\ln(1-f))}{N(N-1)}$$

where pmax is the number of memories that are fixed point of the network's dynamics.

Evaluate p_{max}: testing the stability of pattern μ



$$h_i^b = \sum_{j=1}^{N} W_{ij}\xi_j^{\mu} < fN\theta \ (\text{for i} \setminus \xi_i^{\mu} = 0) \ \ ?$$

$$h_i^f = \sum_{j=1}^{N} W_{ij}\xi_j^{\mu} > fN\theta \ (\text{for i} \setminus \xi_i^{\mu} = 1) \ \ ?$$

Compute $P(W_{ij} = 1 | \xi_i^{\mu} \neq \xi_j^{\mu}) = C_b$ and $P(W_{ij} = 1 | \xi_i^{\mu} = \xi_j^{\mu} = 1) = C_f$

Approximate the distribution of fields $P(h_i^x = S) = \binom{N}{S}(fC_x)^S(1-fC_x)^{N-S}$

Approximate $P(h_i^x = S)$ using Sirling

Approximate : $P(h_i^b > fN\theta) \simeq P(h_i^b = fN\theta)$; $P(h_i^f < fN\theta) \simeq P(h_i^f = fN\theta)$

References

(1) Nadal J-P (1991)
(2) Amit DJ, Fusi S (1994)
(3) Brunel N, Carusi F, and Fusi S (1998)

Contact: alexis.dubreuil@parisdescartes.fr

Learning protocol (H)

In the large N limit, capacity is finite if (2):

$$f = \beta \frac{\ln N}{N} \text{ with } \beta = O(1) \quad \text{and} \quad \delta = \frac{2f(1-f)q_-}{f^2 q_+} = O(1)$$

Evolution of synapses during learning can be describe by a Markov process:

$$\begin{bmatrix} Proba(W_{ij}(\mu+1)=0) \\ Proba(W_{ij}(\mu+1)=1) \end{bmatrix} = \begin{bmatrix} 1-f^2q_+ & 2f(1-f)q_- \\ f^2q_+ & 1-2f(1-f)q_- \end{bmatrix} \begin{bmatrix} Proba(W_{ij}(\mu)=0) \\ Proba(W_{ij}(\mu)=1) \end{bmatrix}$$

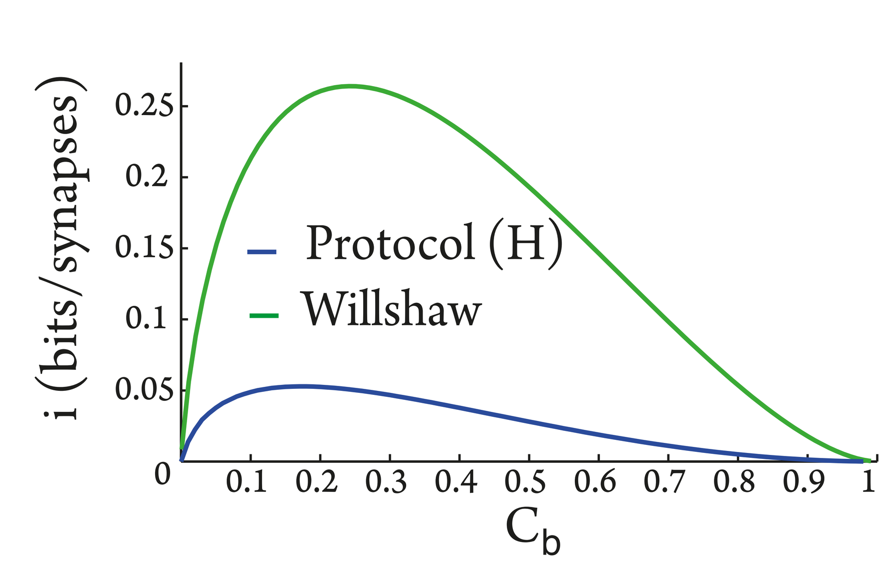
Which allows to compute, in the small f limit for a pattern presented μ_0 steps ago:

$$C_b = \frac{1}{1+\delta} \quad \text{and} \quad C_f = C_b + q_+(1-C_b)\exp\left(-\frac{\mu_0 f^2 q_+}{C_b}\right)$$

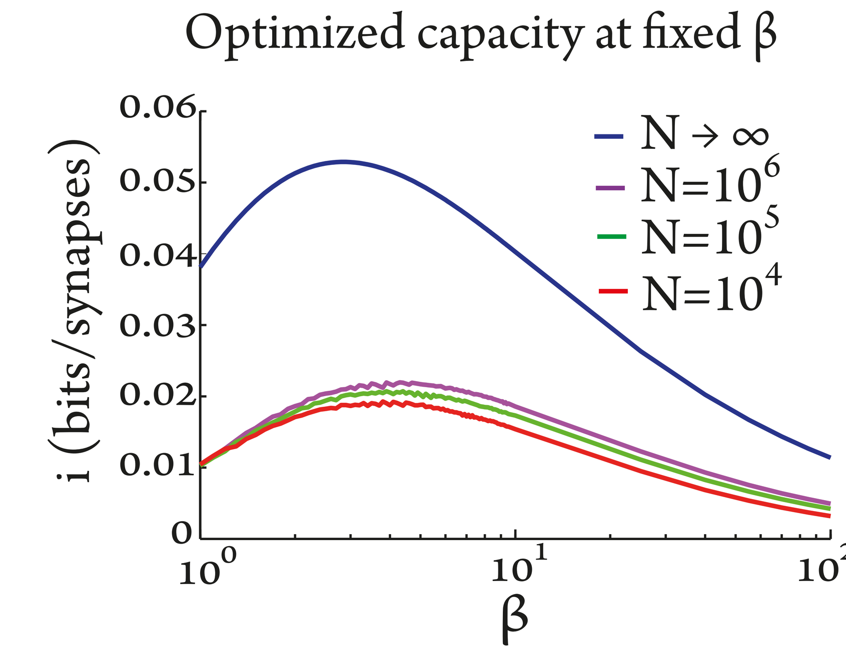
Maximal capacity :

i = 0.053 bits/synapses ; $q_+ = 1$; $C_b = 0.17$; $\beta = 3$

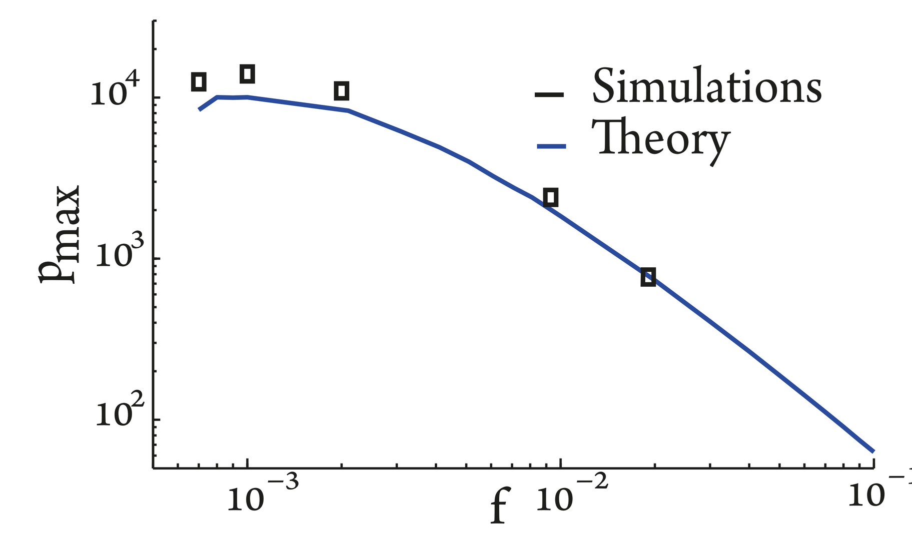
Optimized capacity at fixed Cb

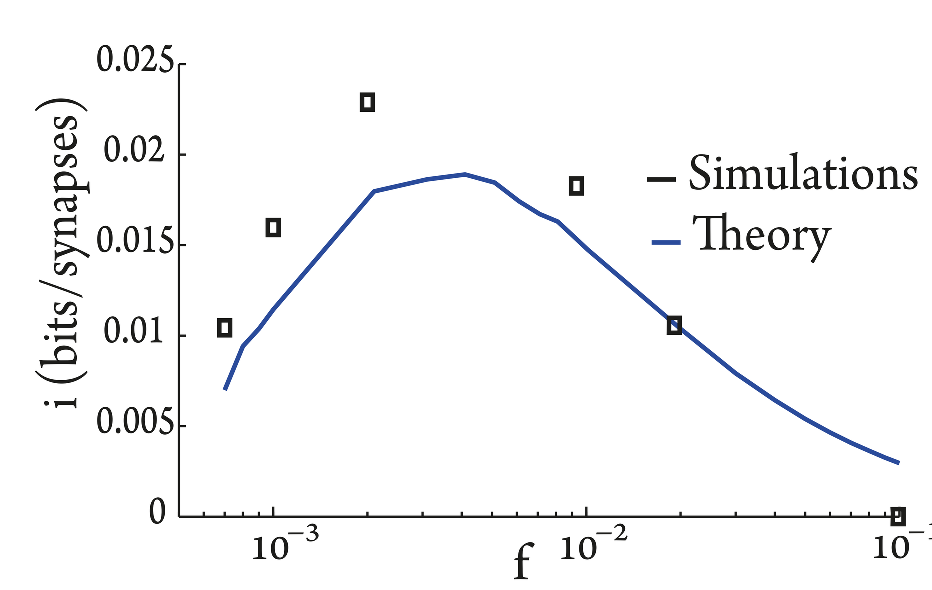


Optimized capacity at fixed β



Comparison with simulations : N=10,000





Learning protocol (C)

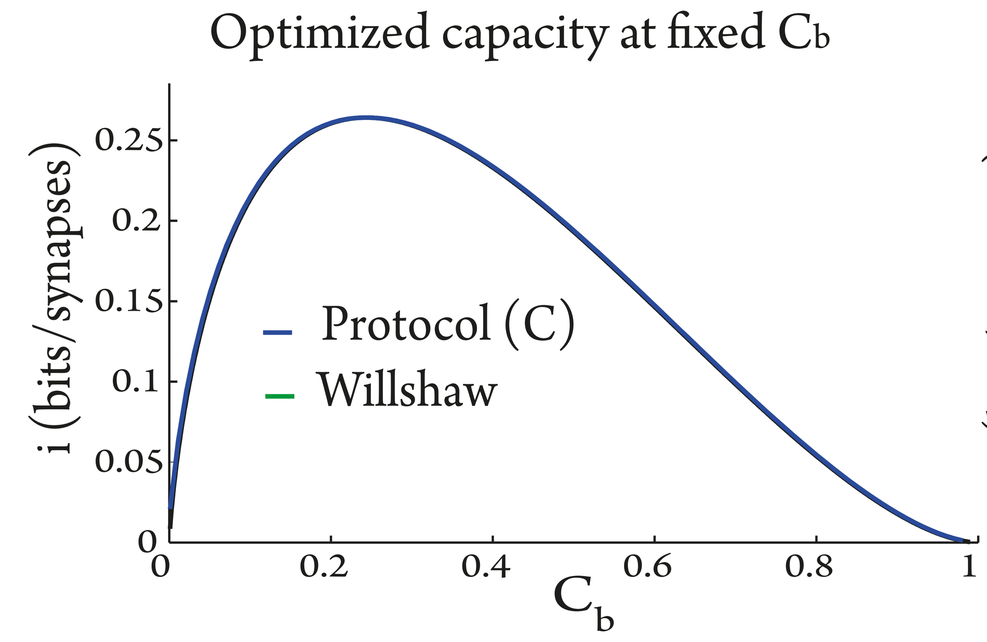
There are the same requirements on f and δ for the storage capacity to be finite.
In the limit of slow learning, $q_+ \ll 1$, one can compute (3):

$$C_b = \sum_{\Pi=0}^{P} \frac{\Pi}{\Pi + \alpha\delta} \frac{\alpha^{\Pi}\exp(-\alpha)}{\Pi!} \quad \text{and} \quad C_f = \sum_{\Pi=0}^{P-1} \frac{\Pi+1}{\Pi+1+\alpha\delta} \frac{\alpha^{\Pi}\exp(-\alpha)}{\Pi!}$$

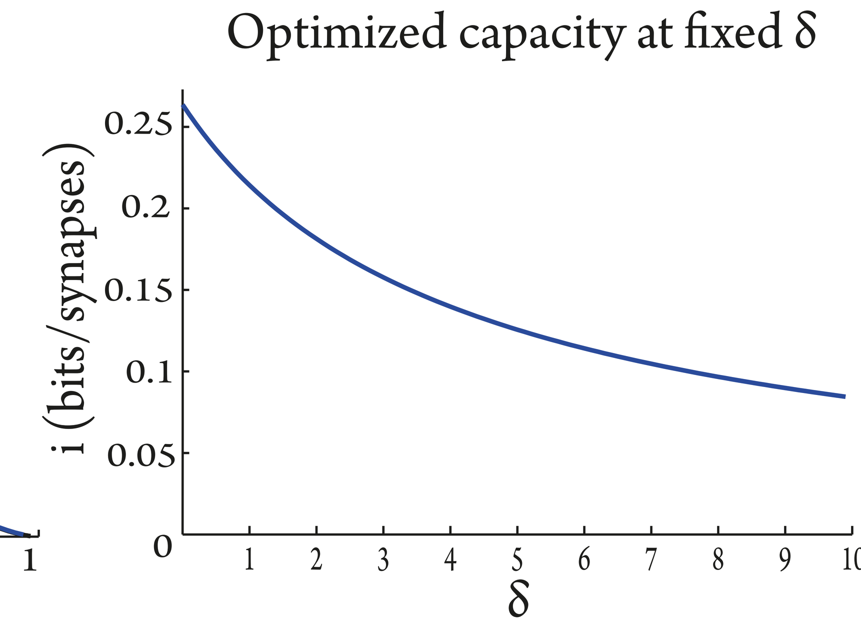
Maximal capacity :

i = 0.26 bits/synapses ; $C_b = 0.24$; $\delta \to 0$

Optimized capacity at fixed Cb



Optimized capacity at fixed δ



Conclusion

- Huge finite-size effects: for realistic network sizes, capacity is less than half the large N limit one. But finite-size effects do not dramatically change dependency on parameters.
- Networks learning slowly through multiple presentations are able to store about 5 times more information than networks able to learn in one shot: this is one of the computational advantage (in terms of storage capacity) of having a slow learning system.